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《Fourier and Real Analysis》

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Here is a syllabus and some descriptions of the course. The course will consists of the two parts.
(i) Fourier Analysis and applications,
(ii) Real Analysis and measure theory.

The course is an introduction to the theory of Fourier series and transform (with some applications) as well as to the theory of Lebesque integration. Fundamental ideas and rigorous proof will be presented. Topics of the course to be covered include Fourier series, their convergence and applications, Poisson kernel, Cesaro and Abel summability, Plancherel formula, Poisson summation formula, measures, measurable sets and functions, Lebesgue integral and Fubini theorem.

There are many books on this subject. We will use two textbooks both written by **Stein and Shakarchi: "Fourier Analysis" and "Real Analysis"**, respectively. Other books that I personally use neither in English not in Chinese. So for some topics some special lecture note will be provided.

Syllabus (tentative and subject to change if any):

1. Diveregent series. Cesaro and Abel summability
2. The Genesis of Fourier Analysis
3. Basic Properties of Fourier Series
4. Convergence of Fourier Series
5. Some applications of Fourier series to different area of mathematics
6. Applications of Fourier series to ODE and PDE
7. Elementary theory of the Fourier transform on R with applications
8. The Poisson summation formula
9. Some elements of the theory of The Fourier Transform on R^d.
10. Applications of Fourier transform to PDE
11. Measurable sets and the Lebesgue measure, measurable functions
12. The Lebesgue integral, Fubini theorem

The grading criteria: the students will have 12–16 homeworks (depending on the progress of students during the class). Completely made homeworks give 30% of the final mark, the colloquium gives also 30%, and the exam gives other 40%.

After completing the course, students should:

1. Know the definition and basic (analytic) properties of Fourier series and Fourier transform, and be able to apply them in different areas of mathematics such as ODE, PDE, functional and integral equations, ergodic theory, geometry etc.
2. Know definition and the basic properties of the Lebesgue integration